At a scale assumed to be representative of a rigid porous medium the mass balance equation used can be written in a general form (see equation 1), for isothermal conditions.

$$\frac{d}{dt}\int m^i dV = \int \vec{J^i}.\vec{n}dS + \int q^i dV \tag{1}$$

Which leads to equation 2 by applying Gauss divergence theorem :

$$\frac{d}{dt}m^i = -div(\vec{J^i}) + q^i \tag{2}$$

Where m represents mass per volume, $\vec{J}$ mass flux, q sinks and sources and i labelling the mass components (water, air). The mass accumulation term and mass fluxes reads:

$$m^i = \phi \sum_\beta S_\beta \rho_\beta X_\beta{}^i \tag{3}$$

$$\vec{J^i} = \sum_\beta X_\beta^i \left[-k\frac{k_{r\beta}\rho_\beta}{\mu_\beta}\left(\vec{\nabla}P_\beta - \rho_\beta\vec{g}\right)\right] - \sum_\beta \rho_\beta D_\beta{}^i \vec{\nabla}X_\beta{}^i \tag{4}$$

Where ϕ, S_β, ρ_β, k, $k_{r\beta}$, μ_β, P_β are the porosity, saturation, density, absolute permeability, relative permeability, dynamic viscosity and pressure of phase β, respectively. $X_\beta{}^i$ is the mass fraction of component i present in phase β and $D_\beta{}^i$ the dispersion tensor, composed of hydrodynamic dispersion and diffusion.

Several hypotheses can be done in order to simplify the system of equations:

- The gaseous phase is the result of the mix between two ideal gas (dry air and water vapor). The mix is also considered as ideal leading to the fact that the total gas pressure is equal to the sum of the pressure of the two gas described before;

- Gravity forces are negligible compared to forces due to pressure;

- Skeleton is incompressible and non-expansive (constant temperature);

- There is no production of air and water during transport (no source terms in the continuity equations 5 and 6).

These assumptions permit to simplify the system of equations as follows:

$$\frac{d}{dt}\left(\phi\left(S_l\rho_l X_l{}^w + S_g\rho_g X_g{}^w\right)\right) = -div(\vec{J^w}) \quad (5)$$

$$\frac{d}{dt}\left(\phi\left(S_l\rho_l X_l{}^a + S_g\rho_g X_g{}^a\right)\right) = -div(\vec{J^a}) \quad (6)$$

$$\vec{J^w} = -X_l{}^w k\frac{k_{r_l}\rho_l}{\mu_l}\vec{\nabla}P_l - X_g{}^w k\frac{k_{r_g}\rho_g}{\mu_g}\vec{\nabla}P_g - \rho_g D_g{}^w\vec{\nabla}X_g{}^w - \rho_l D_l{}^w\vec{\nabla}X_l{}^w \quad (7)$$

$$\vec{J^a} = -X_g{}^a k\frac{k_{r_g}\rho_g}{\mu_g}\vec{\nabla}P_g - X_l{}^a k\frac{k_{r_l}\rho_l}{\mu_l}\vec{\nabla}P_l - \rho_g D_g{}^a\vec{\nabla}X_g{}^a - \rho_l D_l{}^a\vec{\nabla}X_l{}^a \quad (8)$$

Knowing that ϕ and k are known (input of the model) and that ρ_l and μ_l are constant (only dependent of temperature), this eight equations' system (3 equations for flux in 3D) must be solved for 18 unknowns, namely: S_l and S_g, k_{rl} and k_{rg}, P_l and P_g, ρ_g, the mass fraction $X_l{}^w$, $X_l{}^a$, $X_g{}^w$ and $X_g{}^a$ and the fluxes J_w and J_a (3 components for each). Then, several equations have to be added.

The saturations' (see equation 9) and mass fraction' relations (see equation 10 and 11) can be written as follows:

$$S_l = 1 - S_g \quad (9)$$
$$X_l{}^w = 1 - X_l{}^a \quad (10)$$
$$X_g{}^a = 1 - X_g{}^w \quad (11)$$

The relative permeability of liquid can be expressed as a function of liquid saturation, as proposed by Van Genuchten (see equation 12). The gas permeability is deduced from the liquid permeability (see equation 13).

$$k_{r_l} = \sqrt{S_l}\left[1 - \left(1 - S_l^{\frac{1}{\lambda}}\right)^\lambda\right]^2 \quad (12)$$

$$k_{rg} = 1 - k_{rl} \quad (13)$$

The liquid pressure is a function of the capillary pressure, P_c, and the gas pressure (see equation 14) and the capillary pressure can be expressed as a function of liquid saturation, as proposed by Van Genuchten (see equation 15):

$$P_l = P_g - P_c \quad (14)$$

$$P_c = -P_0 \left(S_l^{-\frac{1}{\lambda}} - 1 \right)^{(1-\lambda)} \tag{15}$$

Mass fraction in related to relative humidity, RH, as follows (see equation 16):

$$\frac{X_g{}^w}{X_g{}^a} = HR \Rightarrow X_g{}^w = \frac{HR}{1+HR} \tag{16}$$

where HR is related to capillary pressure, after Kelvin's law:

$$HR = \frac{P_v}{P_{vsat}} = exp\left(\frac{M_w P_c(S_l)}{\rho_l T (T + 273.15)} \right) \tag{17}$$

P_v, P_{vsat} and M_w are vapor pressure, saturated vapor pressure (that is constant at constant temperature) and molecular weight of water, respectively.

Mass fraction of air in liquid phase is related to air pressure, P_a, after Henry's law (see equation 18), K being a constant at constant temperature:

$$X_l{}^a = \frac{P_a}{K} \tag{18}$$

where Dalton's Law of Partial Pressures can be applied (see equation 19):

$$P_g = P_a + P_v \tag{19}$$

Assuming that density of gas is the sum of density of gas components (see equation 20) and applying ideal gas law (see equations 21 and 22), the next 3 equations reads:

$$\rho_g = \rho_a + \rho_v \tag{20}$$
$$P_v M_w = \rho_w RT \tag{21}$$
$$P_a M_a = \rho_a RT \tag{22}$$

where M_w, M_a, ρ_v and ρ_a are the molecular weight of water and air and the densities of vapor and air, respectively.

Finally, the diffusion coefficients for water and air in the gas phase (see equation 23) and the gas relative permeability (see equation 13) were expressed as follows (4 equations):

$$D_g{}^i = \phi \tau_0 k_{rg} S_\beta D_{g0}{}^i \tag{23}$$

with τ_0, the tortuosity and $D_{g0}{}^i$, the diffusion coefficient of the mass component in free gas (without porous media).

Then the system is composed of 26 equations and 26 unknowns (see table 1).

Saturations	2
Gas, air and vapor densities	3
Mass fractions	4
Fluxes	2x3
Relative permeabilities	2
Liquid pressures	1
gas, air and vapor pressures	3
Diffusion coefficients	4
Capillary pressure	1
Total	26

Table 1: Unknowns of the system for liquid and gas transport